Tangential and Angular Joint Estimation Using Polynomial Chaos

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Abstract

Polynomial chaos expresses a probability density function (pdf) as a linear combination of basis polynomials. If the density and basis polynomials are over the same field, any set of basis polynomials can describe the pdf; however, the most logical choice of polynomials is the family that is orthogonal with respect to the pdf. This problem has been studied for the joint estimation of states that are purely translational or angular, and for the independent estimation of mixed states of translational and angular random variables. It is proposed that real valued polynomials that are orthogonal with respect to measures on the unit circle can be used as basis polynomials in a chaos expansion, which would reduce the additional numerical burden imposed by complex valued polynomials.

Motivation

- Noisy measurements, constantly changing environments and imperfect actuators causes uncertainty in dynamical states
- There are many different methods of representing an angular random variable with their own pros and cons
- Jointly estimating angular and translational random variables is a challenge of state estimation
- Polynomial chaos represents a random variable as a coordinate in a polynomial vector space
  - Can estimate any number of moments
  - Doesn't assume the random variable's pdf
  - Can use any orthogonal set as basis polynomials

Example Parameters

Equinoctial element two-body state propagation

$$\lambda_k = \lambda_0 + \sqrt{\frac{\mu}{a^3}} T_k$$

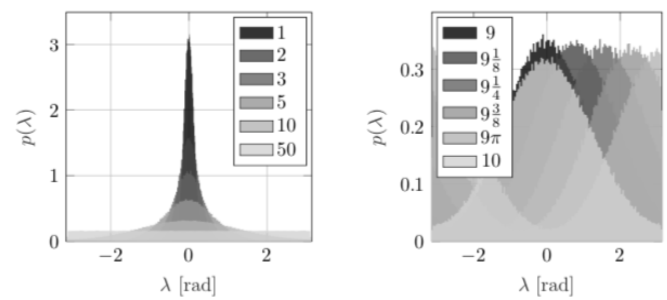
Univariate expansions of mean longitude and semimajor axis

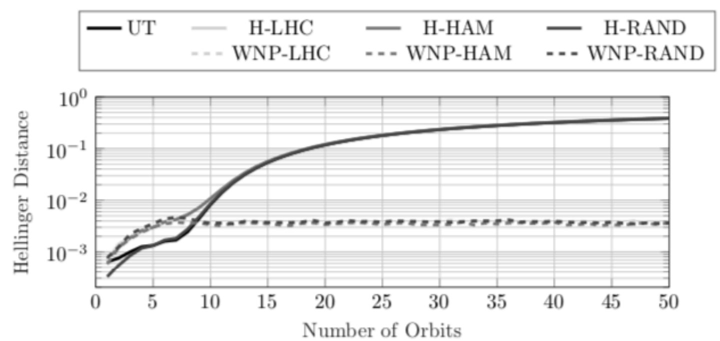
$$\lambda(t,\xi) = \sum_{k=0}^{\infty} \epsilon_{\lambda,k}(t)\Psi_k(\xi) \qquad a(t,\zeta) = \sum_{k=0}^{\infty} \epsilon_{a,k}(t)\Phi_k(\zeta)$$

Initial uncertainty

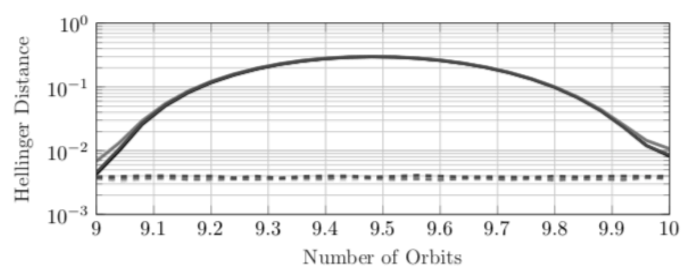
$$\mu_\lambda = 0^\circ \qquad \mu_a = 7444 \text{ km}$$
$$\sigma_\lambda = 1^\circ \qquad \sigma_a = 100 \text{ km}$$

Title





(a) λ at period intervals.



(b) λ across a half period.

Polynomial Chaos

Representation of a random variable

$$\varepsilon(x,\xi) = \sum_{k=0}^{\infty} \epsilon_k(x)\Psi_k(\xi)$$

Random Variable   Chaos Coefficient   Basis Polynomial

Moments from chaos coefficients

$$\mu = \epsilon_0(x) \qquad \boldsymbol{P} = \sum_{k=1}^{\infty} \boldsymbol{\epsilon}_k \boldsymbol{\epsilon}_k^T$$

Wrapped Normal

$$p_w(\theta) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{k=-\infty}^{\infty} \exp\left[\frac{-(\theta-\mu+2\pi k)^2}{2\sigma^2}\right]$$

Common Chaos Types

Type	Polynomial	Hypergeometric Series	Support	PDF
Continuous	Legendre	${}_2F_1$	$[-1,1]$	Uniform
	Jacobi	${}_2F_1$	$[-1,1]$	Beta
	Laguerre	${}_1F_1$	$[0,\infty)$	Exponential
	Prob. Hermite	${}_2F_0$	$(-\infty,\infty)$	Normal
Discrete	Charlier	${}_2F_0$	$\{0,1,2,\dots\}$	Poisson
	Meixner	${}_2F_1$	$\{0,1,2,\dots\}$	Negative Binomial
	Krawtchouk	${}_2F_1$	$\{0,1,\dots,N\}$	Binomial
	Hahn	${}_3F_2$	$\{0,1,\dots,N\}$	Hypergeometric

Conclusions

- Polynomial chaos can be used to quantify angular random variables using real valued polynomials
- Angular and translational random variables can be jointly estimated using polynomial chaos
- When compared with polynomial chaos expansions that treat the angular random variable as its tangent projection, real valued unit circle polynomials provide better results
- Joint state uncertainty quantification can lead to a more complete state esitmate

I would like to acknowledge the Office of Graduate Studies for funding, and Dr. Kyle J. DeMars for advising this research